



## Prof. Dr. Jean-Michel Bismut The hypoelliptic Laplacian - Applications

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### ORT:

Humboldt-Universität zu Berlin, Institut für Mathematik  
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Room 3.001

The hypoelliptic Laplacian, by Jean-Michel Bismut (Orsay)

The hypoelliptic Laplacian is a second order operator acting on the total space  $X$  of the (co)tangent bundle of a compact Riemannian manifold  $X$ , which is supposed

to interpolate in the proper sense between the classical elliptic Laplacian of  $X$  and the geodesic

flow. By elliptic Laplacian, we mean the Hodge Laplacian in de Rham or Dolbeault theory, and more generally the square of a classical Dirac operator.

In this last case, the hypoelliptic Laplacian is the square of a hypoelliptic version of the Dirac operator acting on the enlarged space.

One motivation for this construction is the existence of conserved quantities. Among these, there are the Ray-Singer and Quillen metrics. In the case of locally symmetric space, the full spectrum of the elliptic Laplacian can be shown to be invariant under the deformation.

The hypoelliptic Laplacian is essentially the sum of a harmonic oscillator acting along the fibres of  $X$  and of the generator of the geodesic flow. Its construction is nontrivial, the full machinery of local index theory for the elliptic Dirac operator is needed. Moreover, the construction has to be carefully adapted to the considered geometry.

In these two lectures, I will present the main goals of the theory, explain in detail the construction of the hypoelliptic deformation on the case of the circle, and also for complex manifolds, describe the analytic results obtained by Lebeau and

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ourselves, and give a few applications, which include the evaluation of semisimple orbital integrals on reductive groups. Finally, connections with physics, if any, will be outlined.

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